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Aeroelasticity Research for Turbomachine Applications

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The continuing demand for increased performance in turbine engine turbomachinery has aggravated dynamic problems in the various components, particularly the blading. These problems are generally classified into the categories of either flutter or forced response. Historically, the complexity of the flowfield necessitated the development of empirical flutter and forced-response design techniques. However, such empirical correlations have proven to be inadequate when extrapolated beyond past experience levels. Hence, current research effort, coordinated between industry, government, and universities, is directed toward the development of phenomenologically founded approaches to these problems. Presented herein is an overview of this attack on aerodynamically induced vibrations in turbomachines.

I. Introduction

PERFORMANCE requirements in axial flow turbomachines have dictated higher rotational speeds, thinner airfoils, higher pressure ratios per stage, and increased operating temperatures. These have resulted in dynamic problems influencing the structural integrity of the principal components of the engine, particularly the blading. These aerodynamically induced vibration problems are classified into one of the two general categories to be discussed herein: 1) flutter and 2) forced response.

Under some conditions, a blade row operating in a completely uniform flowfield can enter into a self-excited oscillation known as flutter. The motion is sustained by the extraction of energy from the uniform flow during each vibratory cycle, with the flutter frequency corresponding generally to one of the lower blade or coupled blade-disk natural frequencies. A fundamental parameter common to all types of flutter is the reduced frequency $k=\omega b/V$, where b is the blade semichord, ω the frequency of oscillation, and V the freestream velocity. This parameter indicates the significance of the unsteady phenomena and is proportional to the ratio of the chord of the airfoil to the wavelength of the oscillating wake. It has long been a widely used correlation parameter for flutter. For example, as early as 1945 it was noted that torsional stall flutter of compressor blades could occur for reduced frequency values less than or equal to 0.75. The reduced frequency of compressor and fan blades is on the order of unity. Turbine blades currently have much higher values. However, with the trend toward higher flow velocities, it is approaching that for compressor blades. Also, at these higher velocities, shocks will be present in the turbine blade passages, leading to the possibility of shock-induced flutter.

The outstanding feature of flutter is that high stresses exist in the blading, leading to very short-term, high-cycle fatigue failures. As noted by Heiser,[1] if a portion of a single compressor blade fails due to flutter, the result may be instantaneous and total loss of engine power. Because flutter is encountered over a relatively wide range of operating conditions, i.e., a flutter boundary, it is a problem which must be solved before continuing in the engine development or production phase. This solution often involves rotor blade modifications which decrease engine performance. Hence, the design of flutter free-blading in the development of a gas turbine engine is a very significant problem and is currently receiving a great deal of attention from the turbomachine industry as well as the various concerned government agencies.[2,3]

Destructive aerodynamic forced responses have been noted not only in fans and compressors but also in turbine blading. These failure level vibratory responses occur when a periodic aerodynamic forcing function, with frequency equal to a natural blade resonant frequency, acts on a blade row. Such forcing functions are generated at multiples of the ratio of the natural frequency of the blading to the engine rotational frequency. Responses of sufficient magnitude to fail blades have been generated by a variety of sources including upstream vanes, downstream vanes, distortion, rotating stall, surge, bleeds, mechanical sources, and otherwise unidentified or random sources.

As these forced-response problems occur at distinct operating conditions, the engine development or production phase can often be continued by red-lining the engine, thereby avoiding these critical conditions. However, it should be noted that the number of vibratory fatigue failures attributable to forced-response problems far exceeds those due to flutter and, over the lifetime of an engine, these often prove to be more significant in terms of cost and development time than flutter problems.

II. Flutter

Figure 1 is a typical compressor map schematically showing the boundary locations of the more common types of flutter.

At subsonic relative Mach numbers up to the 0.8-0.9 range, either positive or negative incidence stall flutter may occur. It is the oldest, most common type of flutter and is attributed to separated flow on either the airfoil's suction or pressure surface, caused by operating beyond some critical airfoil angle of attack. Bending, torsion, and coupled modes have occurred when this type of flutter is encountered at part speed in a high-speed fan, and at or near the design speed in a low- or high-pressure compressor. In an unclapped rotor, it is generally unphased at low amplitudes, with the possibility of a constant interblade phase angle existing at the large amplitudes of vibration which occur deep in the flutter region. With a clappered rotor, phasing is enforced by the part-span mechanical ties.

Choking flutter in the bending mode usually occurs at negative incidence angles at a part-speed condition with the blade operating either subsonically or transonically. In a choked flow condition, the blade passage inlet flow is constrained to pass through a decreased stream area. Thus, Mach numbers which are greater than the inlet Mach number can occur in the blade passage, thereby leading to the possible existence of passage shocks which can cause flow separation

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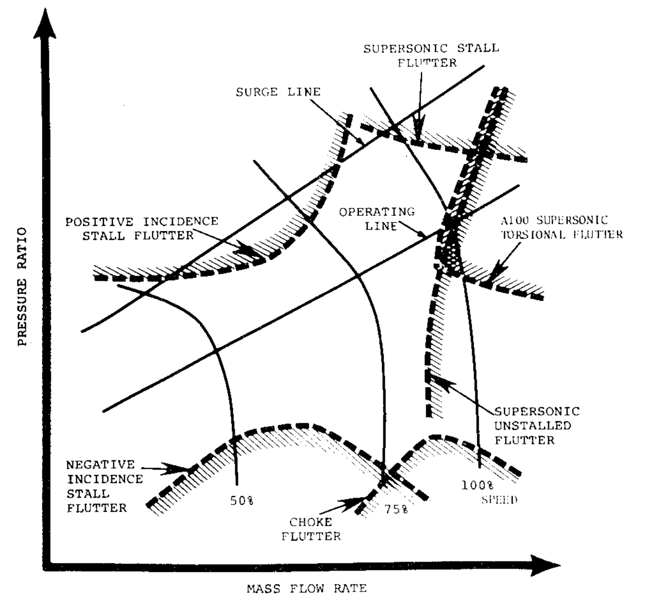

Fig. 1 Schematic of common types of compressor and fan flutter regimes.

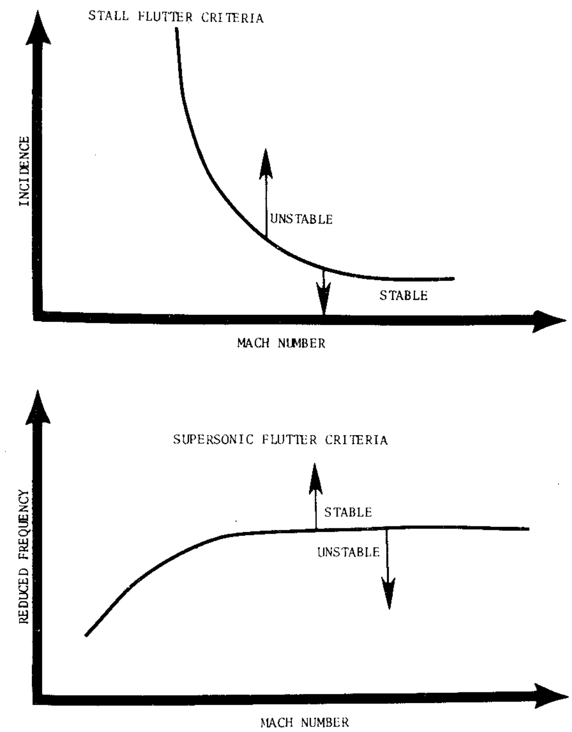

Fig. 2 Typical empirical flutter prediction criteria.

or perhaps couple adjacent blades. The physical mechanism of choke flutter is not fully understood; both high negative incidence angles and choked flow are viable candidates. Analogous to the subsonic stall flutter case, choke flutter is generally unphased at low amplitudes of vibration, with the possibility of a constant interblade phase angle existing at large amplitudes.

Supersonic unstalled flutter can impose a limit on the high-speed operation of the compressor. It has generally occurred in the torsional mode near the operating line where the outer span of the blade operates in a supersonic relative flow with a subsonic axial component. The stresses encountered during this type of flutter can be catastrophically large, with all blades fluttering at a common frequency and a constant interblade phase angle. High-speed operation near the surge line can lead to supersonic stalled flutter, as indicated in Fig. 1. Flutter in the bending mode has generally been associated with these higher pressure ratios. As with unstalled supersonic flutter, a constant interblade phase angle exists in stalled supersonic flutter.

A second type of supersonic torsional flutter encountered near the operating line designated A-100 flutter,[4] also results in catastrophically large stresses with all blades fluttering at a common frequency and a constant interblade phase angle. It is characterized by the existence of a threshold level pressure ratio above which flutter is encountered – below which no flutter is encountered. Increased loading above the threshold pressure ratio can alleviate this flutter, i.e., with increased loading, higher corrected speeds can be attained without encountering flutter.

The overwhelming task of predicting the proper unsteady aerodynamic forces in the various flutter regimes for the cascades of airfoils has necessitated the use of empirical approaches for flutter prediction. These empirical predictions use data acquired during engine and rig test programs. When flutter was encountered, the flutter frequency and interblade phase angle were noted. For each of the flutter regimes encountered, empirical flutter boundaries were then established by correlating the reduced frequency at a specific blade spanwise location vs the relative Mach number or the incidence angle, as schematically depicted in Fig. 2. It should be clearly noted that the flutter data upon which these correlations are based were obtained during exploratory component testing aimed at measuring aerodynamic performance. Hence, minimal attention was spent investigating the flutter, with major effort exerted to eliminate the flutter so that aerodynamic testing could be continued. As a result, these flutter data tend to be haphazard and somewhat random in nature.

Such empirical correlations have become inadequate as advances in technology have required extrapolations beyond past experience levels. For example, in the development of the F-100 engine, serious stall flutter problems were encountered.[4] These failures of empirical prediction rules for blade flutter are largely a result of the inability to properly model the fundamental unsteady aerodynamics. Hence, the current long-range effort, both by industry and government, is being directed toward the development of phenomenologically founded approaches for the prediction of the various flutter phenomena. Short-term empirical efforts are also continuing.

The phenomenological approach is being accomplished by:

1) Establishment of a flutter data bank by obtaining flutter boundary data on rigs instrumented for flutter data as well as detailed blade aerodynamic data.

2) Measurement of the actual unsteady forces acting on a blade during flutter.

3) Development of experimentally verified and directed analytical models of the fundamental phenomena for each type of flutter.

4) Integration of the analytical models into an experimentally verified predictive design system.

Figure 3 presents a schematic of the current overall turbine engine flutter research effort, as detailed by the Air Force Propulsion Laboratory and Office of Scientific Research, NASA Lewis, the Office of Naval Research, and the Naval Air Systems Command.[5]

Full-scale rotating rigs instrumented to establish both the basic aerodynamic and mechanical conditions, as well as the corresponding flutter boundaries, are obviously of primary value. They provide the ultimate in flutter boundary data and also the final test of advanced concepts. For example, the very attractive concept of moving the flutter boundary without compromising the rotor's aerodynamic performance, accomplished by altering the blade structural propeties with composite inlays, is currently being evaluated on a full-scale rig.[6,7] However, such full-scale data are very costly, particularly in a quantity sufficient to provide a minimum data

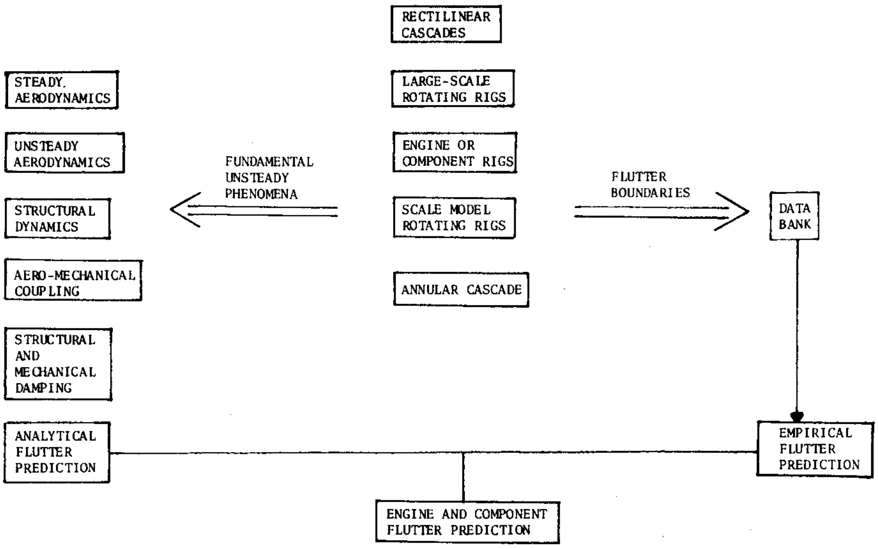

Fig. 3 Schematic of overall turbomachinery research effort.

Table 1 State-of-the-art unsteady cascade analyses

	Flutter		Forced response	
	Subsonic	Supersonic-subsonic axial	Subsonic	Supersonic-subsonic axial
Zero incidence-unstalled				
Zero thickness flat plate	11,[a] 13, 14	16,17,18,19,20	11,13,14	20
Unity pressure ratio				
Incidence-unstalled	12,15	21	31	
Stalled flow				
Zero thickness flat plate	22,23			
Unity pressure ratio				
Full or partially (specified a priori) separated				
Choked flow				

[a] Denotes reference number

bank. Hence, for reasons of cost and geometric flexibility, these tests are being supplemented by scale model efforts and various cascade investigations.

Scale model testing, supplemented by controlled oscillating cascade investigations which simulate a blade section, provide complete flutter and unsteady aerodynamic data for use in a data bank, as well as to verify and/or direct refinements to the various analytical models. However, questions regarding the viable degree of scaling must be resolved. For example, an 8-in. diam model (approximately a 1/5 scale) of an advanced design fan which experienced both positive incidence stall and A-100-type supersonic flutter has been tested.[8] The stall flutter boundaries of the model and rig, both torsion and bending, as well as the steady aerodynamic performance and structural dynamics, exhibited excellent correlation. However, a definite lack of agreement between the model and fan existed with regard to the supersonic flutter.

The annular cascade program[9] is directed toward obtaining flutter boundary data over a range of geometries and operating conditions in the stall and choke flutter regimes. Provided these initial flutter boundaries exhibit good correlation with those of the rotors being modeled, this program should provide a substantial portion of the flutter boundary data needed for the data bank.

All of the preceding testing is involved with measuring flutter boundaries. However, even flutter-free operation of a compressor over its entire map does not guarantee flutter-free operation over the complete airframe flight envelope. One approach to overcoming this shortcoming is to determine the flutter boundary of a compressor by simulating the flow conditions over the complete flight envelope. Currently, this is not possible; however, the Compressor Research Facility being developed at the Air Force Aero-Propulsion Laboratory potentially offers this unique capability.

This type of flutter clearance, although costly and occurring well into the development phase, is certainly necessary. To complement this, however, it would be beneficial to develop a technique to determine a flutter stability margin, analogous to a surge margin. This would permit valuable flutter information to be obtained even when flutter is not encountered, i.e., it would indicate the margin to the various flutter boundaries.

As indicated in Fig. 3, the key elements comprising a flutter stability analysis are steady and time-variant aerodynamic cascade analyses and a vibrational analysis. The output includes the flutter frequency, mode shape, and damping required for stability. The values for structural and mechanical damping then permit the flutter boundary to be predicted, i.e., flutter occurs when the sum of the aerodynamic damping and the mechanical and structural damping is less than or equal to zero.

Structural damping represents the energy dissipated due to cyclic straining of the material, while mechanical damping is the energy dissipated by mechanical interactions, such as blade attachments or interblade shroud interfaces. Currently, values for these quantities are experimentally determined or are estimated based on past experience. No analytical models exist and a strong need for definitive, controlled experiments to establish these models is evident.

The vibration of a bladed disk typical of a rotor is influenced by the geometrical and stiffness properties of both the blades and disk as well as part-span clappers. In such systems, the modes of vibration can be characterized as families of modes with circumferential and diametral nodal lines; the influence of the disk and part-span blade-to-blade ties result in the classic coupled modes.

The accuracy of such analyses is largely dependent on the boundary conditions specified at the part-span shrouds and the blade-disk attachments. To verify the analytical predictions, mode shapes have been measured in stationary rotors using holographic interferometry. Mode shape measurements have also been made in a rotating transonic fan using laser beam reflections from mirrors and diffraction gratings mounted on the blades.[10] However, the experimental mode shape data are not comprehensive as yet. Thus, the immediate concerns of vibration analyses are extensions to account for the behavior of the shroud contacts and of blade root attachments as well as cost reductions in the analysis of blade-disk-shroud systems.

The two-dimensional unsteady aerodynamic cascade analysis is an area of fundamental research interest. Table 1 presents an indication of the state-of-the-art of unsteady cascade analyses and their applicability to the various flutter regimes. For unstalled flutter problems, two-dimensional analyses have been developed which consider: 1) incompressible flow past low-camber, low-incidence cascades[11]; 2) incompressible flow past thick, high-camber cascades[12]; 3) compressible subsonic flow past zero incidence flat plate cascades[13,14]; 4) compressible unsteady flow past thick, cambered airfoils[15]; 5) supersonic flow past flat plate cascades at zero incidence[16-20]; and 6) supersonic high back pressure bending flutter of an actuator disk.[21]

Recently, an unsteady transonic analysis funded by NASA[7] has been undertaken which will consider the effect of the steady flowfield on subsonic-transonic flutter of cascades of cambered, finite-thickness airfoils at an angle of attack.

For stalled flutter problems, analyses are available which consider: 1) incompressible fully separated flow past a flat plate airfoil cascade[22]; and 2) incompressible, partially separated flow past a flat plate airfoil cascade.[23] It is interesting to note that viscous effects and boundary-layer separation are not explicitly included in these inviscid stall flutter models – the separation being specified a priori.

As indicated in Table 1, no fundamental unsteady aerodynamic analyses currently exist which are directly applicable to choke flutter.

For the designer to acquire confidence in the various analytical models as quantitative predictive tools, and to provide guidance to analysts for improvements of their theoretical models, it is imperative to experimentally assess the predictions. The recent development of miniature high-response pressure transducers, and continued improvement and refinements in hot-film and wire sensors, have provided the capability to perform the necessary experiments. These instrumentation developments also offer the possibility of making detailed unsteady aerodynamic measurements on rotating rigs. To obtain quantitative data, however, it is necessary to develop the required experimental techniques and calibration procedures. It should be noted that the interpretation of detailed rotating blade surface dynamic data may well be very difficult, particularly in terms of the two-dimensional theories.

To validate and/or indicate necessary refinements to the time-variant cascade analyses necessary for flutter prediction, fundamental unsteady aerodynamic measurements are being made in controlled oscillating cascades.

Quantitative measurements of the time-variant pressure distribution and its phase relation to airfoil motion have been made for both torsion and translation (bending) mode cascade oscillations in a uniform supersonic inlet flowfield. Correlation of dynamic data obtained for a classical profile airfoil cascade undergoing harmonic torsion[24,25] and translation[26] mode oscillation with predictions obtained from a state-of-the-art flat plate model verified this fundamental model. In particular, the classical profile cascade data and the corresponding predictions exhibited excellent correlation on both the pressure and suction surfaces when the variation in blade-to-blade amplitude of oscillation was considered, as demonstrated in Fig. 4.

One of the fundamental differences between the mathematical model for unstalled supersonic flutter and turbomachinery design practice is the airfoil loading distribution. The model considers zero thickness flat plate airfoils at a unity value for the static pressure ratio. Fan and compressor blades have thickness and camber and operate at pressure ratios greater than unity. To determine the significance of this simplification, the effect of aerodynamic loading on the time-variant aerodynamics of a multiple-circular-arc (MCA) profile airfoil cascade was determined[27] and the results correlated with corresponding flat plate predictions. As seen in Fig. 5, increased static pressure ratio generally results in increased phase lag over the aft cambered portion of the suction surface. The limitations of the flat plate theory are more vividly demonstrated on the pressure surface. On this surface the phenomena are extremely complex as a cambered airfoil surface with two impinging shock waves is involved. Generally, the phase lag data exhibit correlation with the model over the portion of this surface in front of the impinging shock wave locations. The theory does not predict either these intersections or the resulting phase lag, also seen in Fig. 5.

Highly loaded cascades oscillating in an incompressible flowfield near stall are also being investigated.[28] The initial objective of this study is to obtain data describing the motion of the separation point to feed into the analysis of Ref. 23. It should be noted, however, that for this separation point motion to have a first-order effect on the predicted unsteady lift and moment, the magnitude of the motion must be on the order of the blade irrespective of the assumed amplitude of blade motion, an unlikely situation for turbomachine applications. In addition, unsteady force and moment coefficients and aerodynamic work per cycle, as well as indications of the nature and extent of the separation phenomenon, are being experimentally determined over a range of parameters.

Based on the preceding, future analytical effort with regard to unstalled supersonic flutter should be directed toward modeling the effects of airfoil profile and steady aerodynamic loading. With regard to subsonic stall flutter, a partially separated cascade analysis needs to be developed capable of explaining first-order deviations for a fixed separation point analysis. Experimentally, it is highly desirable to increase the reduced frequency of both the unstalled supersonic and subsonic stall oscillating cascades to values which correspond to those of flutter in a rotor. For the case of supersonic flutter, this is being accomplished by oscillating the airfoil cascade at high torsion and translation mode frequencies.[29] To accomplish this, and still maintain two-dimensional mode shapes, high stiffness, minimum inertia airfoils were required, achieved by fabricating these airfoils from graphite/epoxy composite material.

III. Forced Response

The rotor speeds at which forced responses occur are predicted with speed-frequency diagrams which display the natural frequency of each blade mode vs rotor speed and, at the same time, the forcing function frequency vs rotor speed, as indicated schematically in Fig. 6. Wherever these curves cross, forced responses are possible, but no measure can be accurately predicted for the amplitude of the stress.

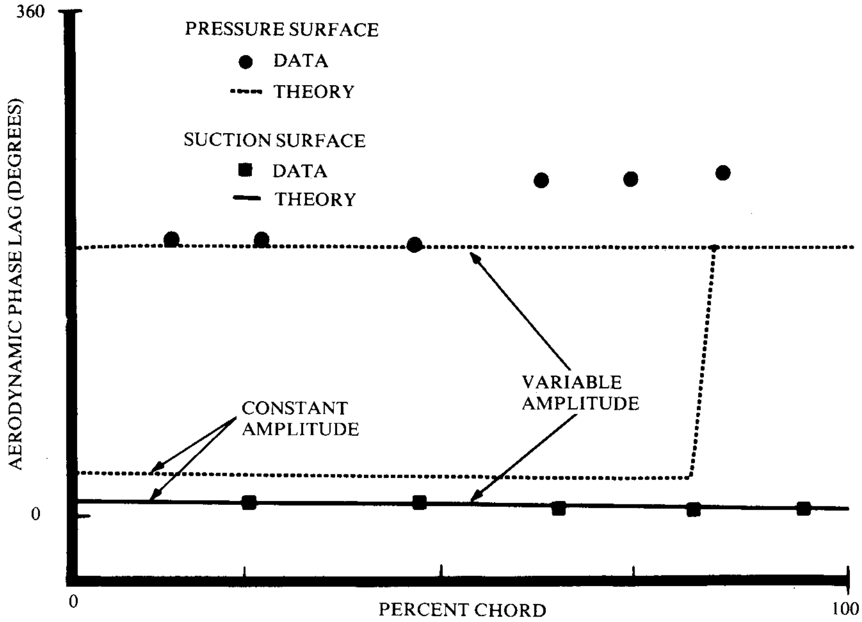

Fig. 4 Classical airfoil cascade torsion mode aerodynamic phase lag data-theory correlation. [25]

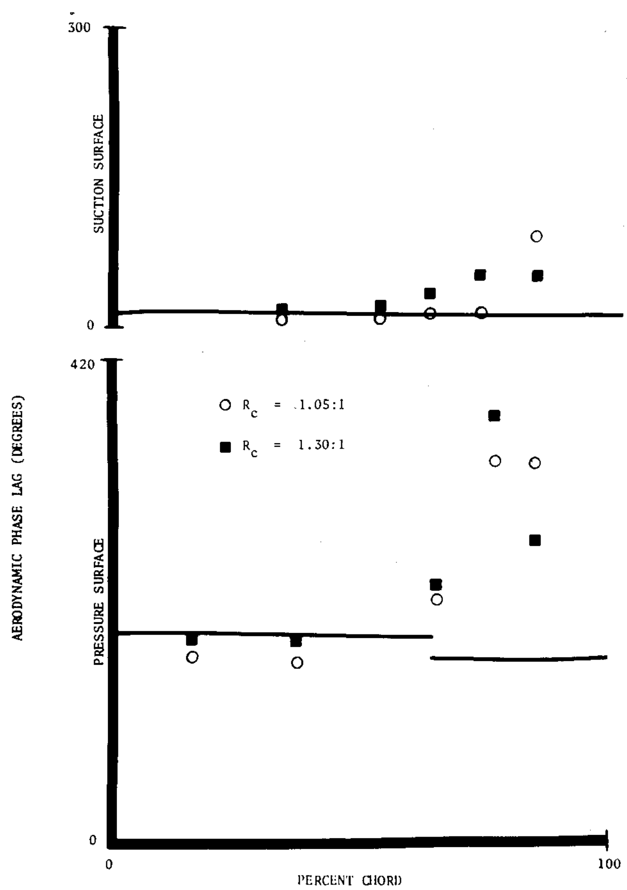

Fig. 5 MCA airfoil cascade torsion mode aerodynamic phase lag data-theory correlation. [27]

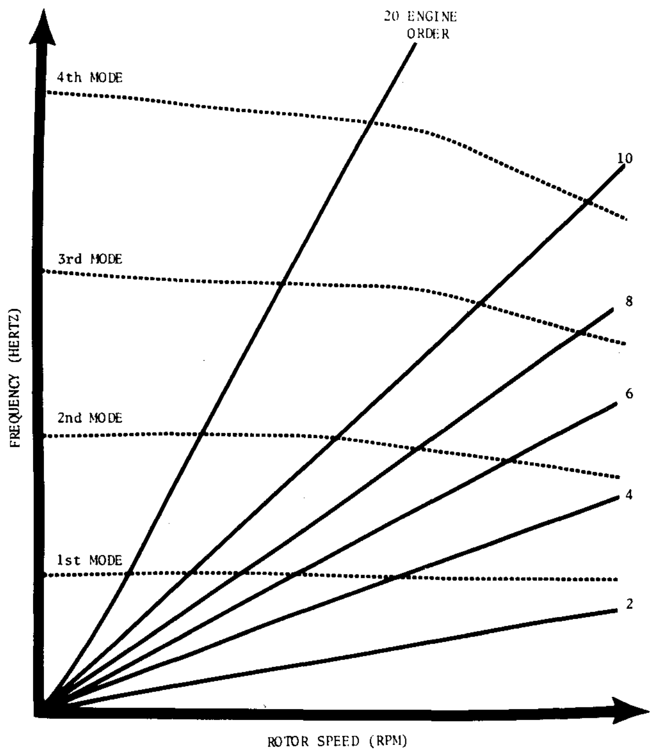

Fig. 6 Typical frequency-speed diagram.

Because it is rarely possible to eliminate all vibration excitation from the operating range of turbomachine blade rows and not possible to predict accurately the amplitude of the stresses with present-day technology, the resonant stresses are not known until the first buildup test of engine components.

One of the primary excitation sources for this type of resonant stress problem is the spatially periodic variations in pressure, velocity, and flow direction in the exit field of an upstream element, which appear as temporally varying in a coordinate system fixed to the downstream blade row. As a result, individual blades are subject to a time-dependent forcing function which can induce high vibratory stresses.

Procedures currently available to predict the aerodynamically forced-response vibratory behavior of a blade row require a definition of the unsteady forcing function in terms of its harmonics. The time-variant aerodynamic response on the blade surfaces to each harmonic of this forcing function is then assumed to be comprised of two parts. One is due to the disturbance being swept past the nonresponding fixed blades. The second arises when the blade responds to this disturbance. The unsteady pressure distribution on the blade surface consists of these two effects. These effects are modeled by means of two analyses. A linearized gust analysis is used to predict the time-variant aerodynamics of the fixed nonresponding blades to each harmonic of the disturbance. An analysis, wherein the blades

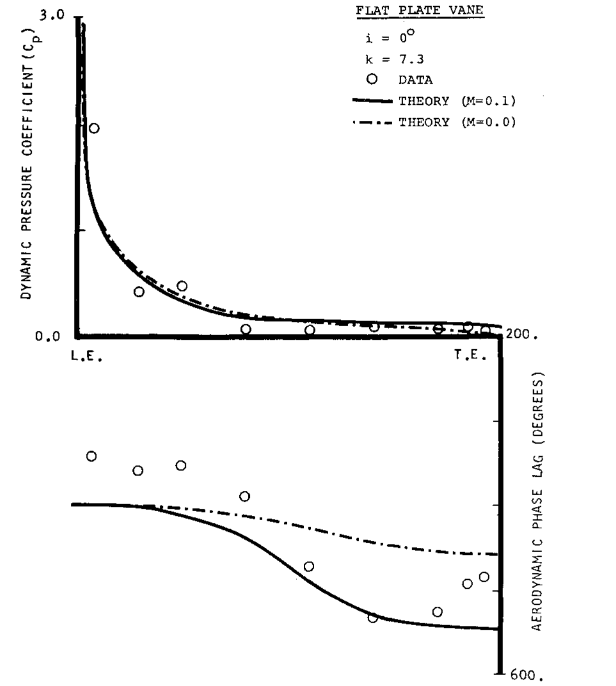

Fig. 7 Correlation of zero incidence flat plate vane row data with incompressible and compressible transverse gust predictions.[34]

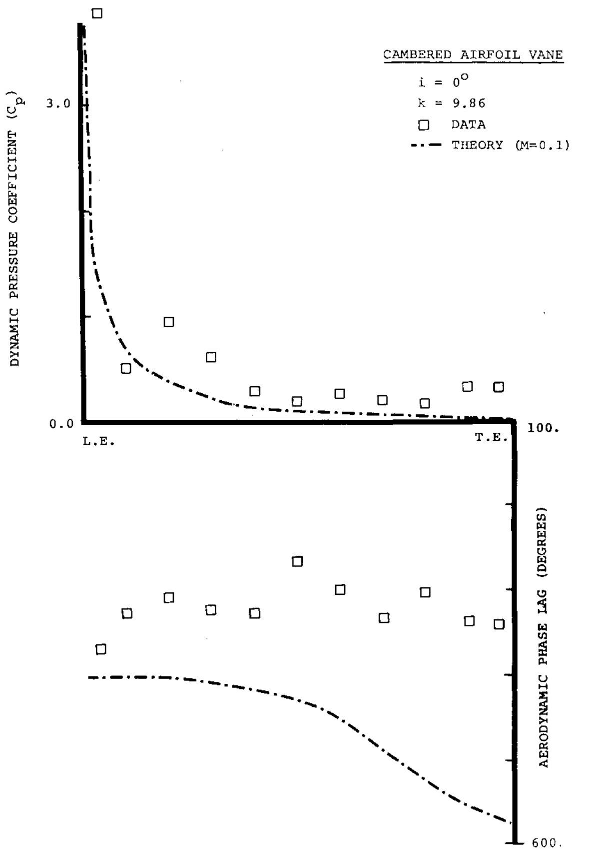

Fig. 8 Correlation of zero incidence cambered airfoil vane row data with the compressible transverse gust predictions.[35]

are assumed to be harmonically oscillating, is then used to predict the additional aerodynamic effect due to the blades responding. Superposition of these two effects can be performed only with knowledge of the modal pattern and amplitude of response of the blading because the magnitude of the pressure field resulting from the blade's motion is dependent upon the amplitude of the motion. Thus, an

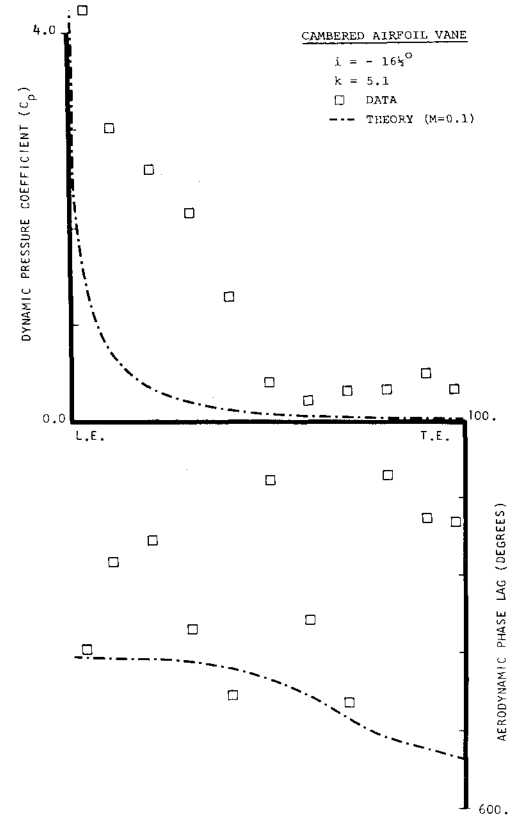

Fig. 9 Correlation of large incidence cambered vane row data with compressible transverse gust prediction.[35]

iterative solution containing the gust analysis, the oscillating blade analysis, and the blade structural dynamic analysis as key elements is necessary to predict the total response of a blade subjected to an upstream-generated spatially periodic disturbance.

The aerodynamic gust analysis is an area of fundamental research interest. Of direct application to turbomachinery design are the unsteady aerodynamic analyses for cascaded airfoils noted in Table 1. Flat plate airfoils moving through a transverse gust in a subsonic compressible flow,[13,14] as well as slightly cambered airfoils in an incompressible flow,[30] have been investigated. Also, the case of finite thickness, camber, and mean incidence blading in an incompressible unsteady inlet distortion is considered in Ref. 31.

There are many mathematical and physical assumptions inherent in these models, yet only a limited quantity of appropriate fundamental experimental data exists with which to assess the range of validity of the models or to indicate refinements necessary to develop a valid predictive design model.

An unsteady inlet flow direction on a single airfoil has been simulated by a Karman vortex street from an upstream transverse cyclinder.[32] The flowfield created by the cylinder has a vertical velocity component which varied in both directions. This is a serious drawback for extension to airfoil cascades as it would result in the velocity direction varying from blade to blade. A subsonic cascade wind tunnel capable of generating variable inlet flow direction has been developed.[33] This system is currently limited to low frequencies of oscillation and hence low reduced frequency values.

The time-variant pressure distribution on a downstream stator vane row generated by upstream rotor wakes has been measured in a large-scale, low-speed research compressor over a range of reduced frequencies and incidence angles. Both classical flat plate[34] and cambered NACA series 65[35,36] vanes were investigated. Figure 7 presents the zero incidence angle flat plate cascade data together with the predictions from Ref. 13. As seen, the data-theory correlation is excellent, thereby indicating the validity of the assumptions in this flat plate analysis. Figure 8 presents the zero incidence cambered airfoil data together with the flat plate predictions. As seen, this data-theory correlation is less than satisfactory over the aft portion of the airfoils where the camber effects become significant. The effect of incidence angle on the time-variant aerodynamics is demonstrated in Fig. 9 for the cambered airfoil vane row. As seen, this correlation is unacceptable due to the unsteady aerodynamic phenomena not modeled in the analysis. Thus, the flat plate data verify the assumptions inherent in the flat plate analysis but, based on the cambered vane data, additional analytical modeling effort is clearly needed. In particular, the data indicate that airfoil camber as well as nonzero incidence angle effects need to be considered.

To significantly impact the development of a valid, experimentally verified, predictive design system, it is necessary to continue the preceding fundamental unsteady aerodynamic experiments and theoretical developments. However, it is also necessary to initiate a unified government-industry-university forced-response research program, analogous to the one outlined herein for flutter. Such a program would include not only research on the fundamental level just described, but would also involve applied research and testing which would yield data in actual rig and engine environments and begin the systematic development of a forced-response data bank.

IV. Summary

A brief overview of the current state-of-the-art of research and development effort directed at the problems of aerodynamically induced vibrations in turbomachines has been presented.

Although the frequency of vibratory fatigue failures resulting from forced responses far exceeds those due to flutter, they are not as immediately overwhelming to an engine development program. As a result, extensive effort has been directed toward the problem of flutter in gas turbine engines, with forced-response problems just beginning to receive attention on a fundamental research level. To impact the development of a phenomenologically based predictive design system, it is necessary to expand the current research effort and initiate an overall unified government-industry-university forced-response research and development program, analogous to that undertaken for flutter.

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